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Phase transition in compact QED$_3$ and the Josephson junction

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Abstract: We study the finite temperature phase transition in 2+1 dimensional compact QED and its dual theory: Josephson junction. Duality of these theories at zero temperature was established long time ago in [1]. Phase transition in compact QED is well studied thus we employ the 'duality' to study the superconductivity phase transition in a Josephson junction. For a thick junction we obtain a critical temperature in terms of the geometrical properties of the junction.

Keywords: Field Theories in Lower Dimensions, Nonperturbative Effects, Duality in Gauge Field Theories.

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Contents

1. Introduction

Through Polyakov's seminal works [2] in 2+1 dimensional compact QED and the spontaneously broken Georgi-Glashow model; we have learned that once the effects of non-perturbative objects (monopole-instantons) are taken into account gauge theory vacuum behaves like a dual superconductor [3] which confines electric charges. Long range order in the vacuum is destroyed by the condensation of instantons (which look like the four dimensional monopoles). Even though 2+1 dimensional model is too simple to describe the 'confinement' problem of realistic QCD, the underlying physics in Polyakov's theory is extremely rich and potentially useful for four dimensional physics. To give one example, it was proposed in [4] that chiral phase transition in QCD resembles to the deconfining phase transition in Polyakov's model. In this paper we shall make an other use of this model.

After Polyakov's work, Hosotani [1] wrote an interesting paper not only demonstrating the 'dual superconductor' picture of the 2+1 dimensional gauge theory vacuum but also refining the notion of a dual superconductor in this context. Namely he showed that compact QED vacuum is dual to the barrier region in a Josephson Junction (JJ) instead of a 'usual' one-piece superconductor. In a JJ a normal barrier (non-superconducting) placed between two superconductors *becomes* superconducting in response to the supercurrents that flow through the barrier [5]. If one inserts a monopole and an anti-monopole pair into the barrier one should observe a linear potential between the pair instead of a logarithmic one. A magnetic flux tube is formed and the barrier region confines the monopoles. One can formulate a duality between the compact QED and the barrier region of a (dual) Josephson Junction. Following Hosotani, we will sketch the details of this duality below but for now we should mention that supercurrents in the JJ correspond to the instantons in compact QED.

Our aim in this paper is to study the finite temperature phase transition of JJ through the above mentioned duality. Strictly speaking we shall be interested only with superconductor-normal metal-superconductor (SNS) junctions with *thick* metal barriers instead of superconductor-insulator-superconductor (SIS) junctions which cannot be made so thick. The computations in compact QED are valid in weak coupling and we shall see that its dual theory (SNS) junction should have quite a thick ($\sim 100\mu m$) barrier.

In the context of finite temperature phase transition it is of extreme importance to make a distinction between compact QED and the spontaneously broken ($SU(2) \to U(1)$) Georgi-Glashow model even though these theories look the same at zero temperature.[1] The latter theory accommodates charged (dynamical) particles (W-bosons) whereas the former does not. Since there are no dynamical monopoles in JJ (monopoles in the barrier are put by hand) ; it can not be dual to Georgi-Glashow model. Hence JJ can be dual to compact QED which has a gauge field defined on a compact interval. As in Montonen-Olive duality [6] electric charges in the JJ are dual to the magnetic charges in compact QED which arise as topological objects. But bearing in mind that in JJ electric charge is dimensionless whereas in 2+1 dimensional compact QED magnetic charge (inverse of the gauge coupling) is dimensionful we will need to clarify the previous statement.

As an example of qualitative and quantitative differences between the Georgi-Glashow model and compact QED it was demonstrated in [7] that the deconfining phase transition in the former model is in the universality class of the Ising model whereas deconfining in the compact QED is that of Berezinskii-Kosterlitz-Thouless type [8, 7] and the actual critical temperatures of these phase transitions are different.

2. Compact QED $\sim$ Josephson junction

Compact QED defined by the (Euclidean) path integral

$$\mathcal{Z} = \int \mathcal{D}\, A_\mu \exp\left\{ -\frac{1}{4g^2} \int d^3x F_{\mu\nu} F^{\mu\nu} \right\} \tag{2.1}$$

with O(2) gauge symmetry has a low energy description in the weak coupling in terms of a massive scalar field χ with the following effective partition function [2]

$$\mathcal{Z}_{eff} = \int \mathcal{D}\, \chi \exp\left\{ -\frac{g^2}{32\pi^2} \int d^3x \{ (\partial_\mu \chi)^2 + M^2 \cos\chi \} \right\}, \tag{2.2}$$

[1] This distinction was not observed in [1] since at zero temperature W-bosons are bound in pairs and do not effect the low energy dynamics.

[2] A proper formulation of the theory can be carried out on the lattice but here we do not wish to dwell on this

where M is the dynamically generated χ-field mass due to the Debye screening of monopole-instantons and parametrically it is related to monopole fugacity. Therefore it is much much smaller than g^2.

The electromagnetic field of the monopole-instantons defined as $H_\mu(x) \equiv \frac{1}{2}\epsilon_{\mu\nu\sigma} \times F_{\nu\sigma}$ is computed to be[3]

$$H_\mu = i\frac{g^2}{4\pi}\partial_\mu \chi(x)\,. \tag{2.3}$$

One can analytically continue the fields to Minkowski space as $H_\mu = (F_{23}, F_{31}, F_{12}) = i(-E_2, E_1, -iH)$ which yields

$$\{H, E_1, E_2\} = \frac{g^2}{4\pi}\left\{\frac{\partial}{\partial t}, -\frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_1}\right\}\chi\,. \tag{2.4}$$

Together with the sine-Gordon equation these are the equations of compact QED and next we turn our attention to the Josephson Junction. When a Josephson junction is connected to a DC source a pair current density J is driven through the barrier:

$$J = J_c \sin\phi\,, \tag{2.5}$$

where J_c is the maximum supercurrent density that the junction can support and ϕ is the phase difference of Landau-Ginzburg wave function in the two superconductors. The astonishing feature of the Josephson junctions occurs in the presence of zero voltage difference. Quantum mechanical nature of the phenomenon provides us a DC current via a constant (not necessarily zero) phase difference. Therefore ϕ, the phase difference of the pair wave function between the two superconductors is the key parameter of the Josephson effect. Assuming that the x^3 direction is parallel to the normal of the barrier one obtains the following equations in the junction [5]

$$\{E_3, H_1, H_2\} = \frac{1}{2e(\lambda_1 + \lambda_2 + d)}\left\{\frac{\partial}{\partial t}, -\frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_1}\right\}\phi\,, \tag{2.6}$$

where d is the thickness of the barrier and λ_1 and λ_2 are the penetration depths of superconductors. Maxwell's equation for JJ yield the sine-Gordon

$$\left(\partial_x^2 + \partial_y^2 - \frac{1}{v^2}\partial_t^2\right)\phi = \frac{1}{\Lambda_J^2}\sin\phi\,, \tag{2.7}$$

where v and Josephson penetration depth Λ_J are given in terms of the properties of the junction and the fundamental constants.

It is evident that both compact QED and JJ are described by similar sets of equations. It was proposed in [1] that these theories are dual given that electric fields in compact QED are replaced by magnetic fields in the Josephson junction.

[3] Our normalization of H_μ is different from the normalization of [1]. We keep the dimension of the electric and magnetic fields to be $mass^2$ both in the compact QED and Josephson Junction

As usual this electric-magnetic duality works only if one exchanges electric-magnetic charges [6]. But one should realize that the electric charge in the Josephson junction is dimensionless but on the other side magnetic charge, which is proportional to $1/g$, in the 3D compact QED is dimensionful. Therefore a naive correspondence between $1/g$ and e is not possible. A proper way to formulate the correspondence of these two theories is to conjecture the following identification

$$\chi = \phi \qquad \text{and} \quad \{H, E_1, E_2\} = \{E_3, H_1, H_2\}\,. \tag{2.8}$$

This identification leads to

$$\frac{g^2}{4\pi} = \frac{1}{2e(\lambda_1 + \lambda_2 + d)}\,. \tag{2.9}$$

We therefore assume that given the above relations compact QED *describes* Josephson Junction. This identification clearly is stronger than the duality of these two theories.

3. Phase transition in Josephson junction

One can assume that the correspondence we have suggested above, which is valid at zero temperature, continues to hold at finite temperature and study the phase transitions in these theories. In compact QED one expects that at a certain temperature instantons are bound in pairs and their effect in the partition function is suppressed. At this point the deconfining sets in and the gauge theory vacuum is ordered. In Josephson Junction above a certain temperature the supercurrents cease to exist and the barrier region looses its 'superconductivity'. The deconfining phase transition in compact QED is better understood [8, 7] and in what follows we will study the phase transition in JJ through the deconfining phase transition of the compact QED. Let us recap briefly what happens in compact QED at high temperature.

At zero temperature monopoles interact with a three dimensional Coulomb interaction but at finite temperature interaction becomes logarithmic at distances larger than the inverse temperature $(1/T)$. This follows from the fact that the path integral is formulated with periodic boundary conditions in the Euclidean time direction which becomes compact at finite temperature. Therefore the magnetic field of an instanton is effectively squeezed to two dimensions when looked from far away. The instanton density, which is proportional to the photon mass is so small that the average distance between the instantons is much much bigger than $1/T$. Therefore one can dimensionally reduce the theory and obtain a 2D sine-Gordon theory

$$\mathcal{L} = \frac{g^2}{32\pi^2 T}(\partial_i \chi)^2 + \frac{M^2 g^2}{16\pi^2 T}\cos\chi\,. \tag{3.1}$$

This lagrangian describes a two dimensional Coulomb gas and it has been extensively studied as an exactly solvable theory. In particular it is well known that it undergoes a Berezinskii-Kosterlitz-Thouless [9] phase transition. At a temperature

$$T_{BKT} = \frac{g^2}{2\pi} \tag{3.2}$$

monopole-anti-monopole pairs bind to form 'molecules'. The conformal dimension of the cosine term is

$$\Delta = \frac{4\pi T}{g^2}\,. \tag{3.3}$$

Therefore above T_{BKT} the interaction term is irrelevant and the (dual) photon becomes massless. Even though we have given a rather cursory account of the story, detailed study [8] shows that deconfining phase transition in compact QED is that of BKT type.

In the Josephson Junction, according to the duality arguments of Hosotani [1], we expect a similar phase transition at T_{BKT}. Namely above this temperature supercurrents in the barrier are not freely flowing or proximity effects are suppressed. In terms of the properties of the Josephson Junction one can compute this temperature making use of the duality equation (2.9). Therefore we obtain a phase transition temperature for the Josephson Junction (at zero external magnetic field).

$$T_{JJ} = \frac{1}{e(\lambda_1 + \lambda_2 + d)}\,. \tag{3.4}$$

This formula can only be valid for clean and sufficiently thick junctions. Taking $d = 100\mu m$ and neglecting $\lambda_{1,2}$ one obtains $T_{JJ} \sim 76K$. Although for high temperature superconductors this temperature is not unreasonable, it is still two orders of magnitude larger than what one obtains from experiments [10]. This discrepancy is not surprising given the simplicity of our approach. We have assumed a perfectly clean, infinitely wide metal and neglected all the complicated physics of finite temperature effects. Strictly speaking one should read our formula as giving an upper limit of the transition temperature.

4. Conclusion

We have studied the phase transitions in Compact QED and the Josephson Junction by making use of the electric-magnetic duality suggested in [1]. Since both theories are described by 2D Coulomb gas at finite temperature the phase transitions are that of BKT type and one can obtain the critical temperatures. Even though our computation is quite simple, having neglected many subtle issues that arise in finite temperature superconductors, we think that compact QED broadly describes the physics of large S-N-S junctions.

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Finally as we have alluded to it above: two dimensional sine-Gordon theory defined by eqn. (3.1) is exactly solvable [see [11] and references therein]. In particular one has the full knowledge of the soliton and breather solutions of the theory (with Minkowski signature) as well as various correlation functions of the theory. One is tempted to make use of this field theory knowledge to understand the physics of the Josephson Junction. In fact there is a vast amount of theoretical and experimental work on solitons (fluxons) [12, 13] which discuss the emergence of solitons in the context of JJ. In this work we have have refrained from discussing solitons since we are considering finite temperature theory where everything is de facto time independent (in fact we are in the Euclidean theory). There are two dimensional 'instanton' (rather than soliton) solutions to the finite temperature theory, eqn. (3.1). But these instantons have infinite action and so they are suppressed in the quantum theory at temperatures until entropy dominates over the action. Near the phase transition temperature these instantons play their role and one should see their effects in the physical observables which depend on the field χ such as the supercurrent etc. Although these issues are worth discussing from a theoretical point of view, in the actual experiments junction irregularities and various losses are quite important to the extend that their effects render the simple sine-Gordon picture insufficient as we have seen from the large value of the predicted phase transition temperature. A proper description should take the losses into account.

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